

# Analytical Estimate of Phase Mixing Time of Longitudinal Akhiezer - Polovin Wave

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Phase mixing and eventual breaking of longitudinal Akhiezer - Polovin wave subjected to a small amplitude longitudinal perturbation is studied analytically. It is well known that longitudinal Akhiezer - Polovin wave breaks via the process of phase mixing at an amplitude well below its breaking amplitude, when subjected to arbitrarily small longitudinal perturbation[Phys. Rev. Lett.108, 125005 (2012)]. Here we analytically show that the phase mixing time (breaking time) scales with  $\beta$ (phase velocity) and  $u_m$ (maximum fluid velocity) as  $\omega_p \tau_{mix} \sim \frac{2\pi\beta}{3\delta}[1/u_m^2 - 1/4]$ , where  $\delta$  is the amplitude of perturbation. This analytical dependence of phase mixing time on  $\beta$ ,  $u_m$  and  $\delta$  is further verified using numerical simulations based on Dawson sheet model.

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## I. INTRODUCTION

Study of space - time evolution of nonlinear oscillations and waves in cold plasmas and their breaking has been an interesting and fundamental topic of research over several decades. Beginning with the pioneering work of Akhiezer and Polovin<sup>1</sup> and Dawson<sup>2</sup> this subject has retained its importance till date not only due to its fundamental academic interest to nonlinear plasma theory, but also because it serves as a useful paradigm to elucidate the underlying physics behind a wide range of physical phenomena, ranging from laboratory based laser/beam plasma interaction experiments<sup>3-11</sup> to some astrophysical phenomena<sup>12,13</sup>, where large amplitude oscillations and waves are usually encountered.

In 1956, Akhiezer and Polovin<sup>1</sup> obtained an exact solution representing purely one-dimensional longitudinal travelling wave in a cold plasma including relativistic mass effects. This travelling wave solution was derived by solving the cold plasma relativistic fluid-Maxwell equations in a stationary wave frame. The maximum amplitude of this wave was shown to be limited by the wave breaking limit which is given by  $\frac{eE_{WB}}{m\omega_p c} = \sqrt{2}(\gamma_{ph} - 1)^{1/2}$ . Here  $\gamma_{ph} = 1/\sqrt{1 - \frac{v_{ph}^2}{c^2}}$  is the Lorentz factor associated with the phase velocity  $v_{ph}$  of the Akhiezer - Polovin wave. In 1989, Infeld and Rowlands<sup>14</sup> presented an exact space - time dependent solution for the relativistic cold plasma fluid- Maxwell equations in Lagrange coordinate. The solution presented by them shows explosive behaviour for all initial conditions except for the one which are needed to excite Akhiezer - Polovin waves. Physically, this explosive behaviour arises due to the relativistic electron mass variation which causes the characteristic electron plasma frequency to acquire spatial dependencies, due to which neighbouring electrons get gradually out of phase and eventually cross causing the wave to break<sup>15-18</sup>. This process of wave breaking through gradual phase mixing is not exhibited by a pure Akhiezer - Polovin wave.

Recently Verma *et al.*<sup>19</sup> constructed longitudinal Akhiezer - Polovin travelling wave solutions from the exact space time dependent solution of Infeld and Rowlands using appropriate choice of initial conditions. It was further shown by the same authors<sup>20</sup> that even a longitudinal Akhiezer - Polovin wave breaks through the gradual process of phase mixing at an amplitude well below its wave breaking limit, when it is subjected to arbitrarily small longitudinal perturbations. It was found through extensive numerical simulations that phase mixing time decreases with increasing  $u_m$ ( for a fixed  $\delta$  ) and increasing  $\delta$ ( for a fixed  $u_m$  ),

where  $u_m$  and  $\delta$  are respectively the velocity amplitudes of the Akhiezer - Polovin wave and the applied perturbation. In this paper we present a weakly relativistic calculation which analytically brings out the scaling of phase mixing time with the parameters of the Akhiezer - Polovin wave (  $u_m$  and  $\beta$ ;  $\beta$  is the phase velocity of the Akhezer - Polovin wave ) and the amplitude of the perturbation  $\delta$ . We have further verified our scaling numerically using a code based on Dawson sheet model<sup>2,21</sup>, which shows a good agreement between numerical and analytical results.

In section II, we briefly describe the Dawson sheet model and construct longitudinal Akhiezer -Polovin travelling wave solution using this model<sup>18</sup>. This derivation although equivalent to that presented in ref.<sup>19, 20</sup> is physically more transparent. It is presented here for the sake of completeness. Section III is devoted to weakly relativistic calculations required for the estimation of phase mixing time of longitudinal Akhiezer - Polovin wave subjected to small amplitude sinusoidal perturbation. In section IV analytical predictions are compared with numerical findings. Finally section V contains discussions and summary of our results.

## II. RELATIVISTIC TRAVELLING WAVE SOLUTION

According to the Dawson sheet model description of a cold plasma, electrons are assumed to be infinite sheets of charges embedded in a cold immobile positive ion background<sup>21</sup>. Evolution of any coherent mode can be studied in terms of oscillating motion of these sheets about their equilibrium positions. Let  $x_{eq}$  and  $\xi(x_{eq}, \tau)$  respectively be the equilibrium position and displacement from the equilibrium position of an electron sheet. In terms of  $x_{eq}$  and  $\xi(x_{eq}, \tau)$  the associated fluid quantities *viz* number density, velocity and electric field can respectively be written as  $n(x_{eq}, \tau) = n_0 / (1 + \partial \xi(x_{eq}, \tau) / \partial x_{eq})$ ,  $v(x_{eq}, \tau) = \dot{\xi}$  and  $E(x_{eq}, \tau) = 4\pi e n_0 \xi(x_{eq}, \tau)$ . Here dot represents differentiation w.r.t Lagrange time  $\tau$ . These expressions can further be represented in terms of Euler coordinates  $(x, t)$  using the transformations  $x = x_{eq} + \xi(x_{eq}, \tau)$  and  $t = \tau$ . Thus, for any given initial condition, once  $\xi(x_{eq}, \tau)$  for a particular sheet is computed as a function of  $x_{eq}$  and  $\tau$ , the problem of evolution of a coherent mode in space and time is essentially solved in principle. This can be obtained by solving the relativistic equation of motion of a sheet which using Gauss's law, may be

written as

$$\frac{\ddot{\xi}}{(1 - \dot{\xi}^2)^{3/2}} + \xi = 0 \quad (1)$$

where  $\tau \rightarrow \omega_p \tau$ ,  $\xi \rightarrow \frac{\omega_p \xi}{c}$ ,  $\dot{\xi} \rightarrow \frac{\dot{\xi}}{c}$ ,  $E \rightarrow \frac{eE}{m\omega_p c}$ ,  $\omega_p$  is the nonrelativistic plasma frequency. Multiplying Eq.1 by  $\dot{\xi}$ , we get

$$\frac{1}{(1 - \dot{\xi}^2)^{1/2}} + \frac{\xi^2}{2} = a(x_{eq}) \quad (2)$$

Here “ $a$ ” corresponds to the total energy of the sheet. Substituting

$$\xi = \sqrt{2(a - 1)} \sin \alpha \quad (3)$$

solution of Eq.1 becomes

$$\tau = \frac{2}{r'} E(\alpha, r) - r' F(\alpha, r) + \Phi(x_{eq}) \quad (4)$$

which gives  $\alpha$  as an implicit function of  $\tau$  and  $x_{eq}$ , where  $r = [(a - 1)/(a + 1)]^{1/2}$  and  $r' = \sqrt{1 - r^2}$ . Eq.3 along with Eq.4 describes the motion of an electron sheet about its equilibrium position for a given set of initial conditions  $\Phi(x_{eq})$  and  $r(x_{eq})$ . The frequency  $\omega$  of an electron sheet is obtained by integrating Eq.2 between two turning points ( $\dot{\xi} = 0$ ) and is given by

$$\omega = \frac{\pi}{2} \frac{r'}{[2E(r) - r'^2 K(r)]} \quad (5)$$

It is evident from Eq.5 that for arbitrary set of initial conditions, “ $\omega$ ” is in general a function of “ $x_{eq}$ ”. Since any coherent mode is made up of a large number of electron sheets oscillating about their equilibrium positions, this spatial dependency of  $\omega$  causes the neighbouring electron sheets to gradually go out of phase with time, which eventually leads to crossing of electron sheet trajectories resulting in singularities in the electron density profile. This is the phenomenon of phase mixing leading to wave breaking. For a sinusoidal initial density profile and for wave like initial conditions, the phenomenon of phase mixing leading to wave breaking is convincingly demonstrated in references<sup>14–16,22</sup>.

As stated in the introduction, in ref.<sup>19</sup> it is shown, that it is possible to choose a special set of initial conditions which excites a propagating solution with phase velocity  $\beta$ , which

does not phase mix and break. This propagating solution is nothing but a longitudinal Akhiezer-Polovin wave. Absence of phase mixing implies, from Eq.5, that “ $a$ ” (energy of an oscillating sheet) should be independent of “ $x_{eq}$ ” and propagation with a fixed phase velocity  $\beta$  fixes the functional form of  $\Phi(x_{eq})$  as  $\Phi(x_{eq}) = x_{eq}/\beta$ . This form of  $\Phi(x_{eq})$  is obtained by choosing  $\xi$  ( hence  $\alpha$  ) to be entirely a function of  $\psi = \omega(t - x/\beta)$ . Thus the initial conditions for exciting a longitudinal Akhiezer-Polovin wave are

$$\xi_{ap}(x_{eq}, 0) = \frac{2r}{r'} \sin \alpha \quad (6)$$

$$\dot{\xi}_{ap}(x_{eq}, 0) = \frac{2r \cos \alpha \sqrt{1 - r^2 \sin^2 \alpha}}{r'^2 \left( 1 + \frac{2r^2}{r'^2} \cos^2 \alpha \right)} \quad (7)$$

along with  $\alpha(x_{eq}, 0)$  implicitly given by

$$\frac{2}{r'} E(\alpha, r) - r' F(\alpha, r) = -\frac{x_{eq}}{\beta} \quad (8)$$

Following Akhiezer - Polovin’s work<sup>1</sup> we now choose  $u_m$  (maximum fluid velocity) and  $\beta$  as independent parameters, instead of  $a$  (or  $r$ ) and  $\beta$ .  $a$  and  $u_m$  are related to each other through Eq.1 and Eq.2 as  $a = 1/\sqrt{1 - u_m^2}$ . In the next section we add a small perturbation to  $\xi_{ap}(x_{eq}, 0)$  and  $\dot{\xi}_{ap}(x_{eq}, 0)$  which leads to phase mixing and subsequent breaking of longitudinal Akhiezer - Polovin wave.

### III. ESTIMATION OF PHASE MIXING TIME

Adding a small amplitude sinusoidal perturbation of amplitude  $\delta$  and wavelength  $k_{ap}$  (same as the longitudinal Akhiezer - Polovin wave) to  $\xi_{ap}$  and  $\dot{\xi}_{ap}$ , we get

$$\xi_{per} = \xi_{ap} - \frac{\delta}{\omega_{ap}} \sin(k_{ap} x_{eq}) \quad (9)$$

$$\dot{\xi}_{per} = \dot{\xi}_{ap} + \delta \cos(k_{ap} x_{eq}) \quad (10)$$

where  $\xi_{ap}$  and  $\dot{\xi}_{ap}$  are the required initial condition for exciting a longitudinal AP wave<sup>19</sup> and  $\xi_{per}$ ,  $\dot{\xi}_{per}$  are the perturbed initial conditions. The perturbed initial conditions are equivalent to adding a small amplitude sinusoidal density perturbation propagating with phase velocity

$\beta$ , to longitudinal Akhiezer - Polovin wave. In the weakly relativistic limit, keeping terms linear in  $\delta$ , the energy associated with an electron sheet becomes

$$a_{per} \approx a + \left[ -\frac{\xi_{ap}\delta}{\omega_{ap}} \sin(k_{ap}x_{eq}) + \delta \dot{\xi}_{ap} \cos(k_{ap}x_{eq}) \right] \quad (11)$$

where  $a \approx 1 + \xi^2/2 + \dot{\xi}^2/2$ . This in turn gives

$$r_{per}^2 \approx r^2 - \frac{2r^2}{a^2 - 1} \delta \left[ \frac{\xi_{ap}}{\omega_{ap}} \sin(k_{ap}x_{eq}) - \dot{\xi}_{ap} \cos(k_{ap}x_{eq}) \right] \quad (12)$$

Finally substituting  $r_{per}^2$  in Eq.5, the frequency of oscillation in the weakly relativistic limit stands as

$$\omega_{per} \approx 1 - \frac{3r^2}{4} + \frac{3r^2}{2(a^2 - 1)} \delta \left[ \frac{\xi_{ap}}{\omega_{ap}} \sin(k_{ap}x_{eq}) - \dot{\xi}_{ap} \cos(k_{ap}x_{eq}) \right] \quad (13)$$

It is clear from the above expression that the frequency of the wave is dependent on the equilibrium position of the electrons which leads to the phenomena of phase mixing<sup>20</sup>. Following Dawsons' argument<sup>2</sup>, the phase mixing time ( $\tau_{mix}$ ) depends on the spatial derivative of frequency as  $\tau_{mix} \approx \pi/2\xi_{max}(d\omega_{per}/dx_{eq})$ . Differentiating Eq.13 w.r.t  $x_{eq}$  and using  $\xi_{max} = 2r/r'$ , the phase mixing time in this case becomes

$$\tau_{mix} \approx \frac{2\pi\beta}{3\delta} \left[ \frac{1}{u_m^2} - \frac{1}{4} \right] \quad (14)$$

It is clear from the above expression that phase mixing time scales directly with  $\beta$ , inversely with  $\delta$  and has a  $u_m^{-2}$  dependence on  $u_m$ .

In the next section we verify these predictions numerically using Dawson sheet simulation.

#### IV. NUMERICAL RESULTS

Using a code based on Dawson sheet model, we numerically verify the process of phase mixing of a large amplitude Akhiezer - Polovin wave perturbed by a small amplitude sinusoidal perturbation. We first load Akhiezer - Polovin type initial condition with a sinusoidal perturbation of amplitude  $\delta$  in a one-dimensional relativistic sheet code containing  $\sim 10000$  electron sheets. Using these initial conditions the equation of motion for each sheet is then solved using fourth order Runge-Kutta scheme. At each time step, ordering of the sheets is

checked for sheet crossing (electron trajectory crossing). Phase mixing time is measured as the time taken by any two of the adjacent sheets to cross over. We terminate our code at this time because the expression for electric field ( $E = 4\pi en_0\xi$ ) used in equation of motion (Eq.1) becomes invalid beyond this point<sup>2,23</sup>.

Fig-1 shows the space time evolution of the electron density profile of the resultant structure. As time progresses, the density profile becomes more and more spiky as energy is irreversibly transferred to higher and higher harmonics. A manifestation of this process is also seen in the Fourier spectrum where we see a broadening of the spectrum as time progresses (Fig-2). Thus the energy which was loaded in the primary mode eventually distributes over higher modes. The interaction of these high “ $k$ ” modes with the particles (sheets) accelerates the particles, causing the initial delta-function momentum distribution to spread. Fig-3 shows that as time progresses, the momentum distribution function gradually spreads generating multi-stream flow; a clear indication of phase mixing leading to breaking. Figure. 4 - 6 respectively show the variation of phase mixing time with respect to  $\delta$ ,  $u_m$  and  $\beta$  for fixed values of the other two parameters. In all the cases points represent the simulation results and the solid line represents our scaling obtained from Eq.14. In all cases, the analytical expression (Eq.14) shows a very good fit to the observed numerical results, thus vindicating our weakly relativistic calculation.

## V. DISCUSSIONS AND SUMMARY

The phenomenon of phase mixing is a manifestation of spatially-dependent plasma frequency<sup>16</sup>. It is well known that large amplitude longitudinal AP wave breaks via the process of phase mixing at an amplitude well below the breaking amplitude for AP wave ( $\sqrt{2}\sqrt{\gamma_{ph}-1}$ ), when subjected to arbitrarily small longitudinal perturbation<sup>20</sup>. We have derived an expression for this phase mixing time which brings out its dependence on  $u_m$ ,  $\beta$  and  $\delta$ . Our weakly relativistic calculation indicates that the phase mixing time scales linearly with  $\beta$ , inversely with  $\delta$  and has  $1/u_m^2$  dependence on  $u_m$ . We have verified our scaling using numerical simulations.

We note here that, the dependence of phase mixing time on  $\Delta$  (density amplitude,  $(\delta n/n_0)_{max}$ ) can be obtained from Eq.14 as  $\tau_{mix} \approx \frac{2\pi\beta}{3\delta} \left[ \left( \frac{1+\Delta n}{\beta\Delta n} \right)^2 - \frac{1}{4} \right]$  by eliminating  $u_m$  using  $n = n_0\beta/(\beta - u)$ . This shows that for  $\Delta \gg 1$ ,  $\tau_{mix}$  is essentially independent of  $\Delta$  and

for  $\Delta \ll 1$ ,  $\tau_{mix}$  scales as  $\sim 1/\delta\Delta^2$ . Fig-7a,7b show the dependence of phase mixing time on  $\Delta$ . We emphasize here that for  $\Delta \ll 1$ , and  $\delta \sim \Delta$ , our expression for phase mixing time exhibits  $\sim 1/\Delta^3$  scaling, in conforming with the results preented in references<sup>14,15,24</sup>.

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## LIST OF FIGURES

1	(Color online) Space-time evolution of the electron density for an Akhiezer - Polovin wave with velocity amplitude $u_m = 0.55$ with perturbation amplitude $\delta = 0.1$ and $\beta = 0.9995$ . . . . .	11
2	(Color online) Fourier spectrum of a Akhiezer - Polovin wave with velocity amplitude $u_m = 0.55$ with perturbation amplitude $\delta = 0.1$ and $\beta = 0.9995$ at different time steps. . . . .	12
3	(Color online) Momentum distribution of a Akhiezer - Polovin wave with velocity amplitude $u_m = 0.55$ with perturbation amplitude $\delta = 0.1$ and $\beta = 0.9995$ at different time steps. . . . .	13
4	(Color online) Analytical (solid) and numerical (circles) scalings of the phase mixing time for a finite amplitude Akhiezer - Polovin wave for $u_m = 0.20$ (4a), $0.55$ (4b) and $\beta = 0.9995$ as a function of perturbation amplitudes ( $\delta$ ) . . . . .	14
	a . . . . .	14
	b . . . . .	14
5	(Color online) Analytical (solid) and numerical (circles) scalings of the phase mixing time as a function of the amplitude of Akhiezer - Polovin wave ( $u_m$ ) in the presence of a finite perturbation( $\delta$ ) = $0.01$ (5a), $0.1$ (5b) and $\beta = 0.9995$ . . . . .	15
	a . . . . .	15
	b . . . . .	15
6	(Color online) Analytical (solid) and numerical (circles) scalings of the phase mixing time as a function of the phase velocity $\beta$ for a fixed amplitude of Akhiezer - Polovin wave $u_m = 0.2$ , $\delta = 0.01$ (6a) and $u_m = 0.1$ , $\delta = 0.1$ (6b) . . . . .	16
	a . . . . .	16
	b . . . . .	16
7	(Color online) Analytical (solid) and numerical (circles) scalings of the phase mixing time as a function of the density amplitude of the Akhiezer - Polovin wave ( $\delta n/n \sim \Delta n$ ) in the presence of a finite perturbation( $\delta$ ) = $0.01$ (7a), $0.1$ (7b) and $\beta = 0.9995$ . . . . .	17
	a . . . . .	17
	b . . . . .	17

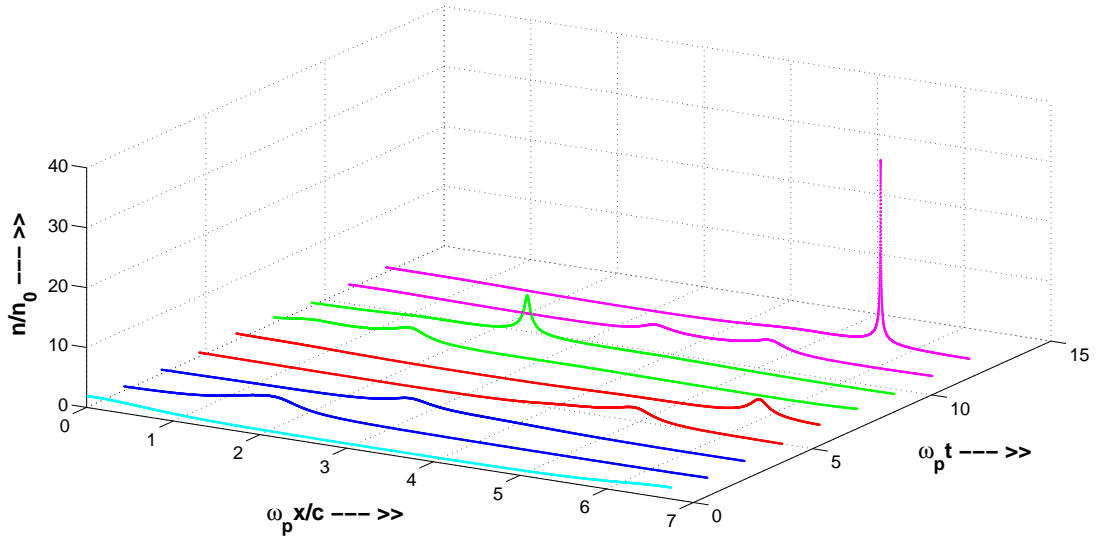


FIG. 1: (Color online) Space-time evolution of the electron density for an Akhiezer - Polovin wave with velocity amplitude  $u_m = 0.55$  with perturbation amplitude  $\delta = 0.1$  and  $\beta = 0.9995$

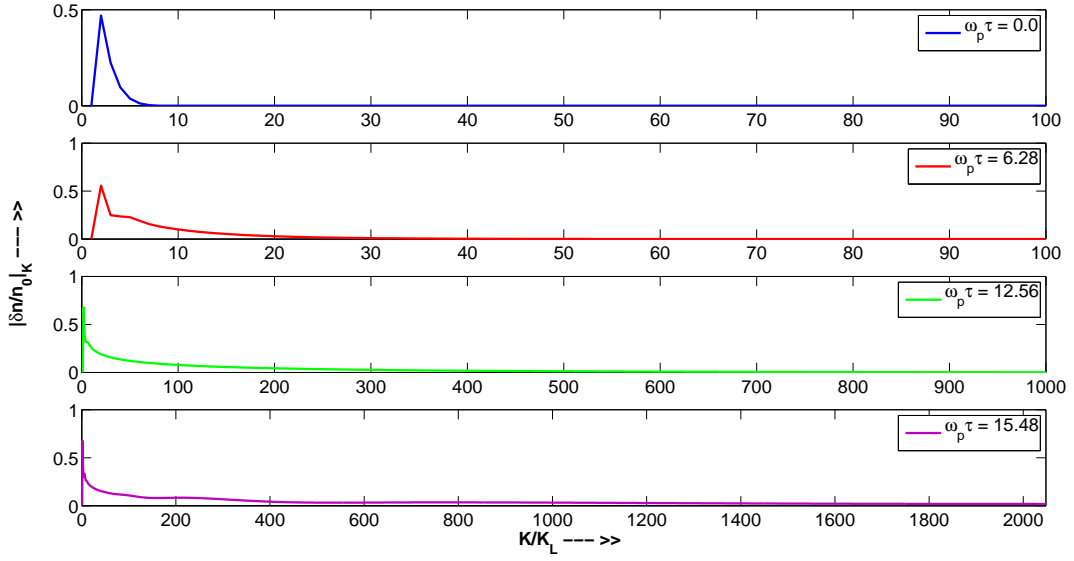


FIG. 2: (Color online) Fourier spectrum of a Akhiezer - Polovin wave with velocity amplitude  $u_m = 0.55$  with perturbation amplitude  $\delta = 0.1$  and  $\beta = 0.9995$  at different time steps.

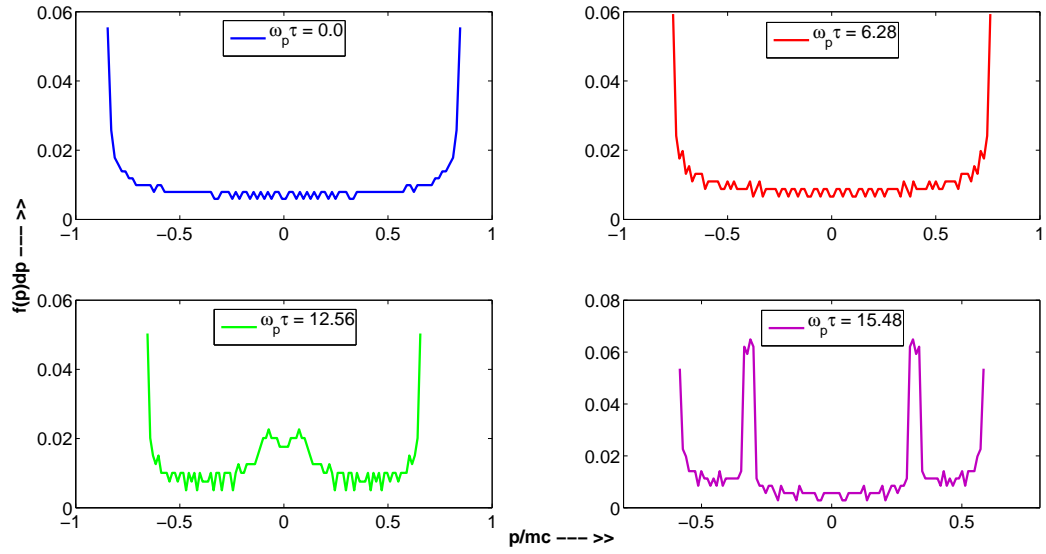
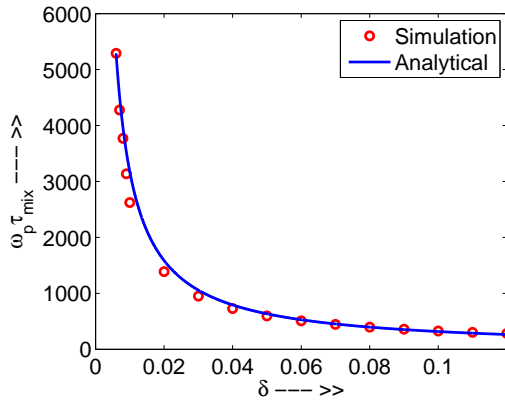
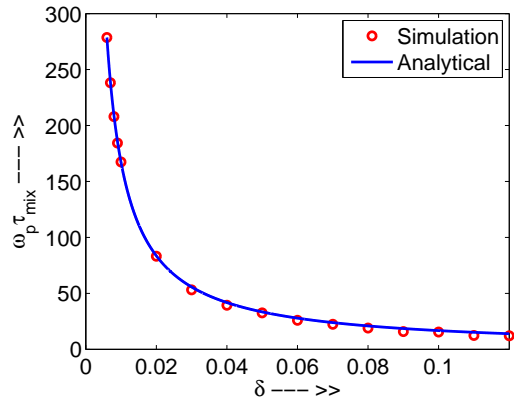


FIG. 3: (Color online) Momentum distribution of a Akhiezer - Polovin wave with velocity amplitude  $u_m = 0.55$  with perturbation amplitude  $\delta = 0.1$  and  $\beta = 0.9995$  at different time steps.



(a)



(b)

FIG. 4: (Color online) Analytical (solid) and numerical (circles) scalings of the phase mixing time for a finite amplitude Akhiezer - Polovin wave for  $u_m = 0.20$ (4a),  $0.55$ (4b) and  $\beta = 0.9995$  as a function of perturbation amplitudes ( $\delta$ )

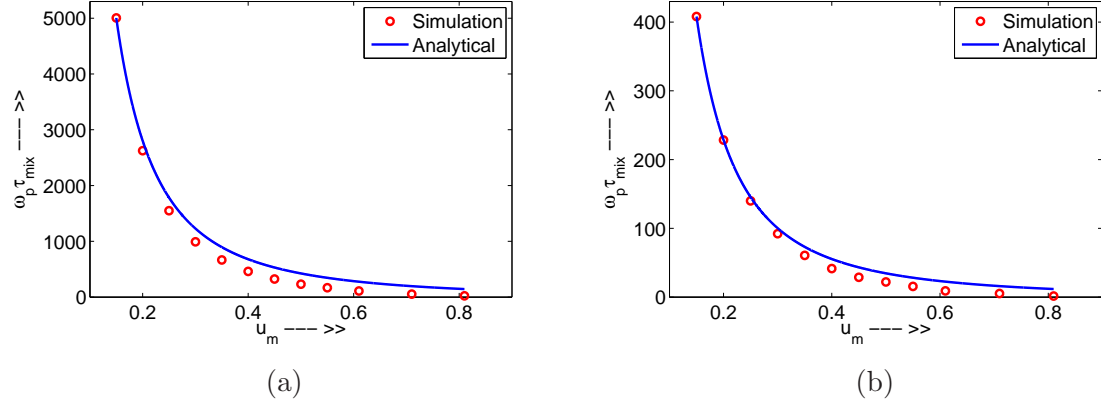


FIG. 5: (Color online) Analytical (solid) and numerical (circles) scalings of the phase mixing time as a function of the amplitude of Akhiezer - Polovin wave ( $u_m$ ) in the presence of a finite perturbation( $\delta = 0.01$ (5a), $0.1$ (5b) and  $\beta = 0.9995$ ).

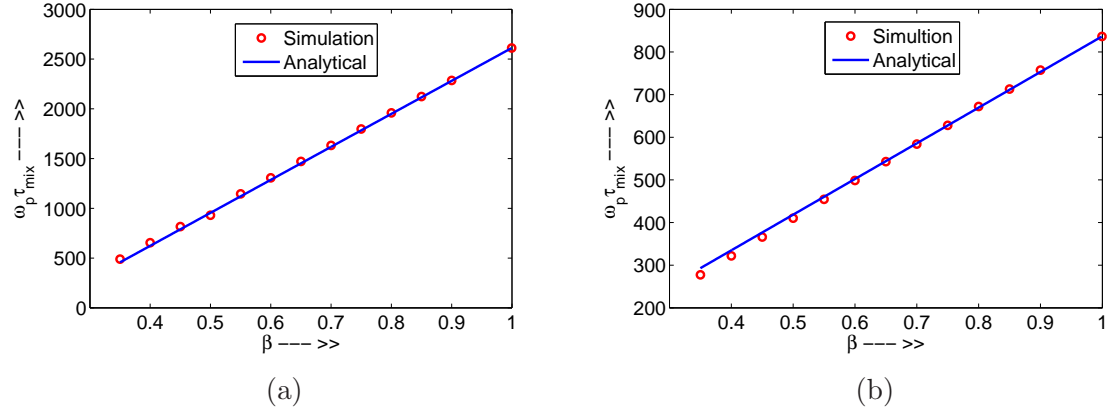


FIG. 6: (Color online) Analytical (solid) and numerical (circles) scalings of the phase mixing time as a function of the phase velocity  $\beta$  for a fixed amplitude of Akhiezer - Polovin wave  $u_m = 0.2$ ,  $\delta = 0.01$ (6a) and  $u_m = 0.1$ ,  $\delta = 0.1$ (6b)



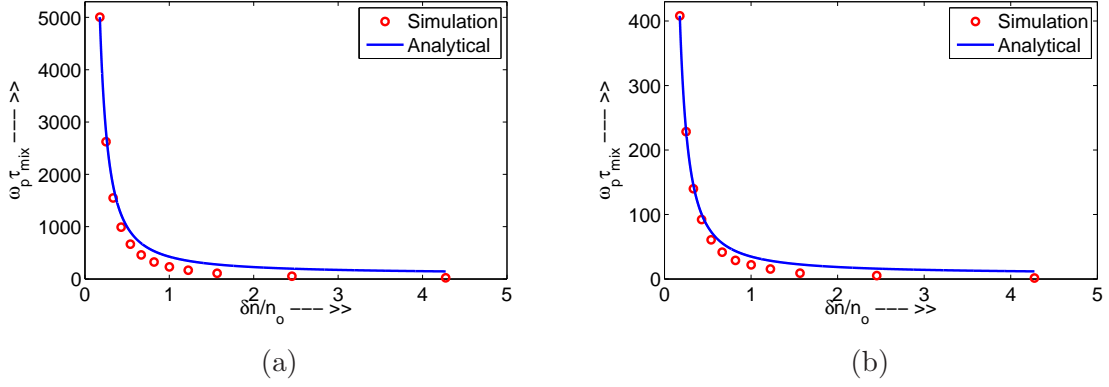


FIG. 7: (Color online) Analytical (solid) and numerical (circles) scalings of the phase mixing time as a function of the density amplitude of the Akhiezer - Polovin wave ( $\delta n/n \sim \Delta n$ ) in the presence of a finite perturbation( $\delta$ ) = 0.01(7a),0.1(7b) and  $\beta = 0.9995$